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Local overlaps, heterogeneities and the local fluctuation dissipation relations

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Abstract

In this paper I introduce the probability distribution of the local overlaps in spin glasses. The properties of the local overlaps are studied in detail. These quantities are related to the recently proposed local version of the fluctuation dissipation relations: using the general principle of stochastic stability these local fluctuation dissipation relations can be proved in a way that is very similar to the usual proof of the fluctuation dissipation relations for intensive quantities. The local overlap and its probability distribution play a crucial role in this proof. Similar arguments can be used to prove that all sites in an ageing experiment remain at the same effective temperature at the same time.

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1. Introduction

Up to now in disordered systems the overlap and its distribution were considered as global quantities, which were defined for the whole system [1]. However, in systems with quenched disorder, it is possible to define (in a non-trivial way) a local overlap that has a point-dependent probability distribution [2]. This new object has remarkable properties that we explore in this paper.

One of the most interesting results is related to the local generalization of fluctuation dissipation relations (FDR) in off-equilibrium dynamics.

It is well known that the FDR in off-equilibrium dynamics are a crucial tool for exploring the landscape of a disordered system [3–6]. These FDR are different from the predictions of the fluctuation dissipation theorem at equilibrium. They can be expressed in a rather simple form that can be easily interpreted from the theoretical point of view. Moreover, the main parameter entering the FDR has a simple interpretation from the point of view of equilibrium statistical mechanics [7–10]. This fact has the consequence that for a given system the form of the FDR is universal in off-equilibrium dynamics, i.e. it is independent of the details of the

dynamics and the way in which the system is put in an off-equilibrium situation (as soon as the system remains slightly out of equilibrium).

In the most studied case, one considers observables that are the average over the whole sample [1, 11–16]. In this case the static–dynamic relations connect the FDR to the static average of global quantities. Recently there have been a few investigations on FDR that involve only given local variables [17–19]. It turns out that using the probability distribution of the single spin overlap, it is possible to give a theoretical foundation to these local FDR and derive the appropriate static–dynamic relations, which involve the probability distribution of the local overlap [2, 18].

The paper is organized as follows. In section 2 we define our main new theoretical tool: the probability distribution of the local overlap; we compute its properties in a few simple cases. In section 3 we recall the main facts about the usual global FDR, while in section 4 we recall the proposed local FDR [2, 18, 19]. In section 5 we derive the local FDR from general principles, prove the appropriate static–dynamic relations and we show that in an ageing regime, in spite of the existence of local heterogeneities, all sites at a given time must be characterized by the same effective temperature. In section 6, before the conclusions, we discuss some methods to compute the probability distribution of the local overlap. Finally, in the appendix, we present some considerations on systems with finite volume.

2. The local overlap distribution

2.1. The formulation of the problem

Our aim is to define the local overlap probability distribution $P_i(q_i)$. Usually the global overlap of two equilibrium configurations (σ and τ) is defined as

$$q = \frac{\sum_i q_i}{N} \tag{1}$$

where $q_i = \sigma_i \tau_i$. For a given sample we can define the overlap probability distribution $P_J(q)$, where J labels the sample. In the glassy phase the function $P_J(q)$ depends on the sample also for very large samples: the global overlap probability distribution $P_J(q)$ is not a self-averaging quantity. The physically interesting quantity is

$$P(q) = \overline{P_J(q)} \tag{2}$$

where the bar denotes the average over the couplings J.

In a different approach [10] one considers the response of the system to the appropriate perturbation and in this way one can define for a given sample a function $P_r(q)$, which for large systems should be self-averaging, i.e. J-independent. This new order parameter distribution codes the thermodynamic responses of the system to random perturbations. According to the principle of stochastic stability [7–10], it should coincide with $P(q)$.

It is evident that the definition of the local overlap must be rather different from that of the global overlap. Indeed, in a naive approach the local overlap of two equilibrium configurations (i.e. $\sigma_i \tau_i$) is always equal to ± 1; if a naive definition is used the probability distribution of the local overlap should be the sum of two delta functions at ± 1: in this way one gets a trivial result.

Moreover, our aim is to define a site-dependent, sample-dependent $P_i(q_i)$ such that

$$P_g(q) = \int \prod_{i=1,N} (P_i(q_i)\, \mathrm{d}q_i)\delta\left(\frac{\sum_i q_i}{N} - q\right) \tag{3}$$

is a self-averaging quantity. In other words $P_g(q)$ cannot coincide with $P_J(q)$ but it should be equal to $P_r(q)$. In the following, we will extend the approach of [10] in order to arrive at a definition of $P_i(q)$ that satisfies the aforementioned properties.

The last requirement is rather important: indeed using the decomposition of the Gibbs state into states (that will be discussed in detail later) we could propose that

$$P_i^n(q_i) \equiv \sum_{\alpha,\gamma} w_\alpha w_\gamma \delta\left(q_i - m_i^\alpha m_i^\alpha\right). \tag{4}$$

We shall not follow this path for three reasons:

- The decomposition of the Gibbs state into states is rather involved; it is better to use it at the heuristic or at the metaphorical level, and define new quantities without using it.
- This definition does not satisfy the requirement of equation (3).
- The quantity $P_i^n(q_i)$ is not related to the local fluctuation dissipation relation; moreover, it is not a *local* quantity as long as it changes considerably when a far away perturbation is introduced. It may be relevant in other contexts, but not here.

2.2. The definition of the local overlap

Let us start from a spin glass sample and consider M identical copies of our sample: we introduce $N \times M$ σ_i^a variables where $a = 1, M$ (eventually we send M to infinity) and N is the (large) size of our sample ($i = 1, N$). The Hamiltonian of this Gibbs system is given by

$$H_K(\sigma) = \sum_{a=1,M} H(\sigma^a) + \epsilon H_R[\sigma] \tag{5}$$

where $H(\sigma^a)$ is the Hamiltonian for a fixed choice of the couplings and $H_R[\sigma]$ is a random Hamiltonian that couples the different copies of the system. A possible choice is

$$H_R[\sigma] = \sum_{a=1,M;i=1,N} K_i^a \sigma_i^a \sigma_i^{a+1} \tag{6}$$

where the variables K_i^a are identically distributed independent random Gaussian variables with zero average and variance 1. In this way, if the original system was d dimensional, the new system has $d + 1$ dimensions, where the contiguous hyperplanes are randomly coupled. We can consider other ways to weakly couple these M-systems; for example another possibility is given by

$$H_R[\sigma] = \sum_{a,b=1,M;i,j=1,N} K_{i,j}^{a,b} \sigma_i^a \sigma_j^b \tag{7}$$

where the variables K are identically distributed independent random Gaussian variables with zero average and variance $(NM)^{-1}$.

As we shall see later the form of H_R is not important: its task is to weakly couple the different hyperplanes that correspond to different copies of our original system. The first choice (equation (6)) is the simplest to visualize and it is the fastest for computer simulations, and the last choice (equation (7)) is the simplest one to analyse from the theoretical point of view. In the following, we do not need to assume any particular choice.

Our central hypothesis is that all intensive self-average quantities are smooth functions of ϵ for small ϵ. This hypothesis is a kind of generalization of stochastic stability. According to this hypothesis the dynamical local correlation functions and the response functions will go uniformly in time to the values they have at $\epsilon = 0$.

We now consider two equilibrium configurations σ and τ in the case of non-zero ϵ and let us define for given K the site-dependent overlap

$$q_i(\sigma,\tau) = \frac{\sum_{a=1,M} \sigma_i^a \tau_i^a}{M}. \tag{8}$$

We define the K-dependent probability distribution $P_i^K(q)$ as the probability distribution of the previous overlap. If we average over K at fixed ϵ, we can define

$$P_i^\epsilon(q) = \overline{P_i^K(q)} \tag{9}$$

where the bar denotes the average over K. We finally define

$$P_i(q) = \lim_{\epsilon\to 0} P_i^\epsilon(q) \tag{10}$$

where the limit $\epsilon \to 0$ is done *after* the limits $M \to \infty$ and $N \to \infty$ (alternatively we keep ϵM and ϵN much larger than 1).

In order to be consistent with the usual approach, we should have that if we define

$$q_t = \frac{\sum_{i=1,N} q_i}{N} \tag{11}$$

the probability distribution $P_t(q)$ of q_t should be self-averaging (i.e. J-independent in the infinite volume limit) and it should coincide with the function $P(q)$ that is the average over J of $P_J(q)$:

$$P_t(q) = P_g(q) \equiv \overline{P_J(q)}. \tag{12}$$

This crucial relation will be proved in the next section.

In a nutshell the construction is rather simple. We consider M weakly coupled copies of the original lattice and in the limit M going to infinity we can define *local* thermodynamic averages. We will assume that we also remain in the infinite volume limit. A discussion of what happens for finite N but $M = \infty$ will be presented in the appendix.

2.3. Some heuristic considerations

What is the rationale of this baroque construction? The heuristic idea is simple. For finite N and given J we can approximately decompose the usual Boltzmann expectation value $\langle\cdot\rangle$ into different states labelled by α:

$$\langle\cdot\rangle = \sum_\alpha w_\alpha \langle\cdot\rangle_\alpha. \tag{13}$$

If we define the state-dependent magnetizations

$$m_i^\alpha = \langle\sigma_i\rangle_\alpha \tag{14}$$

we have that the overlap among two states is

$$q^{\alpha,\gamma} = \frac{\sum_{i=1,N} m_i^\alpha m_i^\gamma}{N} \tag{15}$$

and that the usual overlap distribution is given by

$$P_J(q) = \sum_{\alpha,\gamma} w_\alpha w_\gamma \delta(q^{\alpha,\gamma} - q). \tag{16}$$

The average over J gives the usual $P(q)$.

However, we do not want to average over the J; we want to stick to a given system. In order to make a sensible self-averaging definition we have to consider an ensemble of systems, therefore we will consider the original system plus a small random perturbation ϵH^R. However, if the expectation value of the random perturbation in the various states $\left(E_\alpha^R \equiv \langle H^R \rangle_\alpha\right)$ is much larger than 1, the new w will be given by

$$w_\alpha^R(\epsilon) = \frac{w_\alpha \exp\left(-\beta\epsilon E_\alpha^R\right)}{\sum_\alpha w_\alpha \exp\left(-\beta\epsilon E_\alpha^R\right)} \tag{17}$$

and the states will be completely reshuffled. The quantity E_α^R is typically of order $\epsilon N^{1/2}$, so that it is very large when N goes to infinity at fixed ϵ.

In a first approximation, we can assume that the set of states remains the same after the perturbation also in the limit N goes to infinity at fixed ϵ, i.e. there should be a one to one correspondence between each state before and after the perturbation. However, the quantities $\exp\left(-\beta E_\alpha^R\right)$ have a huge range of variation and therefore those states that dominate the thermodynamics before the perturbation have a new w of order $\epsilon(-AN^{1/2})$ and are thermodynamically irrelevant after the perturbation. In other words if we consider the overlap of the system at $\epsilon = 0$ and $\epsilon \neq 0$, defined as

$$q(\epsilon) = \sum w_\alpha^R(\epsilon) w_\alpha q_{\alpha,\alpha} \tag{18}$$

we find that $q(\epsilon)$ is discontinuous at $\epsilon = 0$, i.e.

$$q(0) > \lim_{\epsilon \to 0} q(\epsilon). \tag{19}$$

The way in which the subset of thermodynamically relevant states changes depends on the distribution of the w_α. However, the principle of stochastic stability tells us that we get the *same* function $P(J)$ if we do the average over the random perturbation at fixed J or over the ensemble of the J in the absence of the random perturbation. Generally speaking, stochastic stability implies that the properties of the system computed at a given value of ϵ are smooth functions of ϵ and that the two limits $N \to \infty$ and $\epsilon \to 0$ do commute. This is a rather deep statement if one considers equation (19).

At small non-zero ϵ the system of M copies of the original system (i.e. the Gibbs system) can be considered a single system. As usual we can assume that the Gibbs system is a state A with a probability w_A and the state A of the Gibbs system is characterized by the fact that the ath subsystem remains in the state α_a (the set of the states α is the same as that at $\epsilon = 0$). In other words there is a one to one correspondence among the states of the Gibbs system and all the possible functions α_a ($a = 1, M$), where as usual α labels the states of the original system.

By changing the variables K we change the weights w_A. We can thus write

$$q_t = \sum_{A,C} w_A w_C \frac{\sum_{a=1,M} q_{\alpha_a,\gamma_a}}{M}. \tag{20}$$

It is clear that we have to prove that this way of generating the weights A and C is such that the distribution of q_t satisfies equation (12). This will be shown in the next section.

2.4. Two explicit cases

Let us consider here the previous construction in two cases where we can perform the relevant computations in an explicit way.

The first case we consider is one step replica symmetry breaking. Here the original system may remain in states labelled by an index α. Each state is characterized by a free energy f_α

and the probability of finding a state in the interval $[f, f + \mathrm{d}f]$ is given by

$$\mathcal{N}(f) = \exp(m\beta(f - f_0)). \tag{21}$$

The equilibrium state of the Gibbs system may be decomposed as

$$\langle\cdot\rangle = \sum_{\alpha} w_A \langle\cdot\rangle_A. \tag{22}$$

The equilibrium state of the Gibbs system is characterized by the weights w_A. These states will have the same distribution of probability (equation (21)). As we have seen, each state A is characterized by variables α_a that are different for each state. Let us consider two states of the Gibbs system (A and C) that correspond to the variables α_a and γ_a (for $a = 1, M$). Therefore, we can compute their overlap as

$$q_i^{A,C} = \frac{\sum_{a=1,M} \sigma_i^{\alpha_a} \tau_i^{\gamma_a}}{M} = \frac{\sum_{a=1,M} m_i^{\alpha_a} m_i^{\gamma_a}}{M} = \frac{\sum_{a=1,M} q_i^{\alpha_a,\gamma_a}}{M} \tag{23}$$

for $A \neq C$ and

$$q_i^{A,A} = \frac{\sum_{a=1,M} \sigma_i^{\alpha_a} \tau_i^{\alpha_a}}{M} = \frac{\sum_{a=1,M} \left(m_i^{\alpha_a}\right)^2}{M} = \frac{\sum_{a=1,M} q_i^{\alpha_a,\alpha_a}}{M} \tag{24}$$

where σ_i^{α} (or σ_i^{α}) is a generic equilibrium configuration of the state α. We have used the notation

$$q_i^{\alpha,\gamma} = m_i^{\alpha} m_i^{\gamma} \tag{25}$$

and m_i^{α}, the magnetization at the site i in the state α:

$$m_i^{\alpha} \equiv \langle\sigma_i\rangle_{\alpha}. \tag{26}$$

In the limit $M \to \infty$ we can use the law of large numbers and find with probability 1 that

$$q_i^{A,C} = \overline{q_i^{\alpha,\gamma}} \equiv q_i^0 \qquad q_i^{A,A} = \overline{q_i^{\alpha,\alpha}} \equiv q_i^1 \tag{27}$$

where the bar denotes the average over all the states (α and γ). The key observation in the proof of these results is that for a given state A the variables α_a are correlated: for each value of a, a random state is selected.

We finally find that

$$P_i(q) = m\delta\left(q - q_i^0\right) + (1 - m)\delta\left(q - q_i^1\right). \tag{28}$$

In other words, the construction we have used performs automatically the average over all the possible states. It is remarkable that in this case the value of m is constant all over the system, and the site variability is present only in the values of q_i^0 and q_i^1.

The same computations can be done in the case where replica symmetry is broken in two steps. In this case the states can be clustered into families, each state is labelled by two indices (e.g. c, γ or d, δ); the first index labels the family and the second index labels the state in the given family. In this case we have two free energy distributions of the form given in equation (21), one for the states (characterized by parameter m_s) and one for the families (characterized by parameter m_f), where $m_s > m_f$. In the same way as before we find

$$P_i(q) = m_f\delta\left(q - q_i^0\right) + (m_s - m_f)\delta\left(q - q_i^1\right) + (1 - m_s)\delta\left(q - q_i^2\right) \tag{29}$$

where

$$\overline{q_i^{c,\gamma;d,\delta}} = q_i^0 \qquad \overline{q_i^{c,\gamma;c,\delta}} = q_i^1 \qquad \overline{q_i^{c,\gamma;c\gamma}} = q_i^2. \tag{30}$$

In the previous formulae we have assumed that $c \neq d$ and $\gamma \neq \delta$.

This construction has some points of similarities with that introduced in [23]; however, it differs in some crucial points. In this approach we have two kinds of quantities: the weights that are global quantities, and the magnetizations that depend on the point.

For simplicity let us restrict our further analysis to the one step replica symmetry breaking. In this case for each point we can reconstruct the probability distribution of q_i if we know the probability distribution $\mathrm{d}P_i(m)$ of finding a magnetization m_i at the site i in a generic state:

$$q_i^0 = \left(\int \mathrm{d}P_i(m) m \right)^2 \qquad q_i^1 = \int \mathrm{d}P_i(m) m^2. \tag{31}$$

It may be interesting to note that on the Bethe lattice, in the cases where one step replica symmetry breaking is exact, the probability distribution $P_i(m)$ depends only on the local environment (the coupling of the nearby points) and it can be computed in the large N limit by solving local equations [24, 25], suggesting that the probability distribution $P_i(q)$ depends on the local environment. This result will be proved in full generality in the next section.

3. The global fluctuation dissipation relations

In this section we will find a short summary of the main results concerning the global fluctuation dissipation relations.

The usual equilibrium fluctuation theorem can be formulated as follows. If we consider a pair of conjugated variables (e.g. the magnetic field and the magnetization), the response function and the spontaneous fluctuations of the magnetization are deeply related.

Let us call $R_{eq}(t)$ the integrated response (i.e. the variation of the magnetization at time t when we add a magnetic field from time zero) and $C_{eq}(t)$ the correlation between the magnetization at time zero and time t. The fluctuation dissipation theorem implies that we have $R_{eq}(t) = \beta(C_{eq}(0) - C_{eq}(t))$, where $\beta = (kT)^{-1}$ and $3/2k$ is the Boltzmann–Drude constant. If we eliminate the time and plot R_{eq} parametrically as a function of C_{eq}, we have that

$$-\frac{\mathrm{d}R_{eq}}{\mathrm{d}C_{eq}} = \beta. \tag{32}$$

The previous relation can be considered as the definition of the temperature and it is a consequence of the zeroth law of thermodynamics.

The generalized FDR can be formulated as follows in an ageing system. Let us suppose that the system is carried from high temperature to low temperature at time zero and it is in an ageing regime. We can define a response function $R(t_w, t)$ as the variation of the magnetization at time t when we add a magnetic field from time t_w; in a similar way $C(t_w, t)$ is the correlation between the magnetization at time t_w and at time t.

We can also define a function $R_{t_w}(C)$ if we plot $R(t_w, t)$ versus $C(t_w, t)$ by eliminating the time t (this function is interesting in the region $t > t_w$ where the response function is different from zero). The FDR state that for large t_w the function $R_{t_w}(C)$ converges to a limiting function $R(C)$. We finally define

$$-\frac{\mathrm{d}R}{\mathrm{d}C} = \beta X(C). \tag{33}$$

One finds that $X(C) = 1$ for $C > C_\infty = \lim_{\to\infty} C_{eq}(t)$, and $X(C) < 1$ for $C < C_\infty$. The shape of the function $X(C)$ gives important information on the free energy landscape of the problem, as discussed at length in the literature.

It has been shown that in a stochastically stable system the function $X(C)$ is related to the basic equilibrium properties of the system. Let us illustrate this point by considering for definitiveness the case of a spin glass.

Spin glasses are characterized by the presence of a quenched disorder (i.e. the coupling J among spins). For a given probability distribution of J, there are quantities that do not depend on the particular (generic) realization of J and are called self-averaging: the response function and the correlation of the total magnetization belong to this category. In contrast, there are other quantities that depend on the choice of J. A typical example of a non-self-averaging quantity is given by the probability distribution of the overlap. For a fixed value of J, given two equilibrium configurations σ and τ, we define the global overlap as

$$q(\sigma, \tau) = \frac{\sum_{i=1,N} \sigma_i \tau_i}{N} \tag{34}$$

where N is the total number of spins. Let us suppose that there is a magnetic field (albeit infinitesimal) such that the overlap is positive, otherwise the overlap should be defined as the absolute value of the previous expression.

The probability of distribution of q is $P_J(q)$ and it depends on J. The function $P(q)$ is defined as the average of $P_J(q)$ over the different choices of the coupling J and obviously depends on the probability distribution of the variable J.

It is convenient to introduce the function $x(q)$ defined as

$$x(q) = \int_0^q P(q')\,\mathrm{d}q' \tag{35}$$

or equivalently

$$P(q) = \frac{\mathrm{d}x\,(q)}{\mathrm{d}q}. \tag{36}$$

Obviously $x(q) = 1$ in the region where $q > q_{EA}$, where q_{EA} is the maximum value of q where $P(q)$ is different from zero.

The announced relation among the dynamic FDR and the static quantities is simple

$$X(C) = x(C). \tag{37}$$

We shall see later that this basic relation can be derived using the principle of stochastic stability that asserts that thermodynamic properties of the system do not change too much if we add a random perturbation to the Hamiltonian. All that is well known. In the next section we shall see how to play the same music with local variables.

4. The local fluctuation dissipation relations

There are recent results that indicate that the FDR relation and the static–dynamic connection can be generalized to local variables in systems where a quenched disorder is present and ageing is heterogeneous [20, 17–19]. We shall see that these findings need a more general framework to be explained.

For *one given large sample* in the ageing regime, we can consider the local integrated response function $R_i(t_w, t)$, which is the variation of the magnetization at time t when we add a magnetic field at point i starting at time t_w. In a similar way the local correlation function $C_i(t_w, t)$ is defined to be the correlation of the spin at point i with itself at two different times (t_w and t). Quite often in systems with quenched disorder, ageing is very heterogeneous: the functions C_i and R_i change dramatically from one point to another.

It has been observed in simulations [18, 19] that local FDR seem to hold

$$-\frac{\mathrm{d}R_i}{\mathrm{d}C_i} = \beta X_i(C) \tag{38}$$

where $X_i(C)$ has also strong variations with the site. It has also been suggested that in spite of this strong heterogeneity, if we define the effective $\beta_i^{\rm eff}$ at time t at the site i as

$$-\frac{{\rm d}R_i(t_w,t)}{{\rm d}C_i(t_w,t)} = \beta X_i(t_w,t) \equiv \beta_i^{\rm eff}(t_w,t) \tag{39}$$

the quantity $\beta_i^{\rm eff}(t_w,t)$ does not depend on the site. In other words, a slow thermometer [21, 22] coupled to a given site would measure (at a given time) the same temperature on the site independently: different sites are thermometrically indistinguishable.

The reader may be confused: we said that $\beta X_i(C)$ fluctuates and $\beta_i^{\rm eff}(t_w,t)$ does not; as both quantities are equal the two statements seem contradictory. However, what effectively depend strongly on the site are the local correlations and the associated responses but not the fluctuation–dissipation ratio (or equivalently the effective temperature). The effective temperature, as a function of time, does not depend on the site, but the same quantity, as a function of the correlation, does depend on the site because the correlation is a site-dependent quantity.

These empirical results call for a general theoretical explanation [17]. The aim of this paper is to show that these results are a consequence of stochastic stability in an appropriate contest and that there is a local relation among statics and dynamics. In the next section we will define in an appropriate way the local probability distribution of the overlap for a given system at point i (i.e. $P_i(q)$). We will also define the function $x_i(q)$ as

$$x_i(q) = \int_0^q P_i(q')\,{\rm d}q' \tag{40}$$

and show that the static–dynamic connection for local variables is very similar to that for global variables and is given by

$$X_i(C) = x_i(C). \tag{41}$$

In order to prove the local FDR, that will be presented in the next section, it is convenient to reconsider the definition of the correlation function $C_i(t_w,t)$. It is clear that $C_i(t_w,t)$ cannot be measured by observing only one single history of our sample: $\sigma_i(t_w)\sigma_i(t_w+t) = \pm 1$. The correlation function is obtained by averaging over all the possible histories, i.e. by repeating the experiment M times and eventually sending M to infinity. In other words, the two time local correlation functions are not self-averaging quantities as far as histories are concerned.

If we want to define the correlation in such a way that it can be measured by observing a single history, we have to consider the Gibbsian system introduced in section 2. We can consider M identical copies (or clones) of our sample, and the Hamiltonian in this Gibbs system is given by

$$H_0(\sigma) = \sum_{a=1,M} H(\sigma^a). \tag{42}$$

For a given history M-replicated systems, the correlation function can be defined as

$$C_i(t_w,t) = \frac{\sum_{a=1,M}\sigma_i^a(t_w)\sigma_i^a(t)}{M} \tag{43}$$

and in the limit $M \to \infty$ of this quantity is self-averaging, i.e. it will be the same in the entire history of the system. It is evident that the M systems are independent so that the average of one Gibbsian system is equivalent to the average of M usual systems: it corresponds to repeating the same experiment (or computer run) M times. A similar procedure can be done for the response function.

5. Perturbing the system

5.1. Global perturbation

As usual, we can obtain an expression for the probability distribution of the overlaps in terms of the response of the system to an external perturbation. Let us consider for simplicity the effect of adding to the Hamiltonian $H_K \equiv H_0 + H_R$ of the Gibbsian system (where different replicas are already coupled) an extra perturbation given by

$$\Delta H^{(1)} \equiv \sum_{i=1,N,a=1,M} h_i^a \sigma_i^a \tag{44}$$

where the variables h are Gaussian random variables with zero average and variance δ.

By integration by parts, we find that for given K

$$\frac{\langle \Delta H^{(1)} \rangle}{NM} = \frac{\delta\beta \sum_{i=1,N,a=1,M} \left(1 - \langle \sigma_i^a \rangle^2\right)}{NM} = \delta\beta \int \mathrm{d}q\, P_t^\delta(q)(1-q) \tag{45}$$

where $P_t^\delta(q)$ is the function $P_t(q)$ (i.e. the probability distribution of q_t) in the presence of the Hamiltonian $\Delta H^{(1)}$. If we assume that the limit $\delta \to 0$ is smooth, we have the relation

$$\chi^{(1)} \equiv \lim_{\delta\to 0} \frac{\langle \Delta H^{(1)} \rangle}{\delta NM} = \beta \int \mathrm{d}q\, P_t(q)(1-q) \tag{46}$$

for the susceptibility χ^1. The proof is identical to the standard one. On the other hand at $\epsilon = 0$, stochastic stability implies that

$$\chi^{(1)} = \beta \int \mathrm{d}q\, P(q)(1-q) \tag{47}$$

where $P(q)$ is the usual J average of the J-dependent probability distribution.

It is a trivial task to generalize the proof to the other susceptibilities. For example, if we define

$$\Delta H^{(2)} \equiv \sum_{i=1,N,a=1,M,k=1,N,b=1,M} h_{i,k}^{a,b} \sigma_i^a \sigma_k^b \tag{48}$$

where the variables h are Gaussian random variables with zero average and variance $\delta/(NM)$, we get

$$\chi^{(2)} \equiv \lim_{\delta\to 0} \frac{\langle \Delta H^2 \rangle}{\delta NM} = \beta \int \mathrm{d}q\, P_t(q)(1-q^2) = \beta \int \mathrm{d}q\, P(q)(1-q^2). \tag{49}$$

In this way we can compute all the moments of both functions $P_t(q)$ and $P(q)$ and they coincide. The two functions are equal.

5.2. Local perturbation

We can now repeat the same steps as before but locally. Let us consider for simplicity the effect of adding to the Hamiltonian $H_K(\sigma)$ an extra perturbation given by

$$\Delta H_i^{(1)} \equiv \sum_{a=1,M} h^a \sigma_i^a \tag{50}$$

where the variables h are Gaussian random variables with zero average and variance δ. By integration by parts we find that for given K

$$\frac{\langle \Delta H_i^{(1)} \rangle}{M} = \frac{\delta\beta \sum_{a=1,M} \left(1 - \langle \sigma_i^a \rangle^2\right)}{M} = \delta\beta \int \mathrm{d}q\, P_i(q)(1-q). \tag{51}$$

Therefore, we have the relation

$$\chi_i^{(1)} \equiv \lim_{\delta\to 0} \frac{\left\langle \Delta H_i^{(1)} \right\rangle}{\delta M} = \beta \int \mathrm{d}q\, P_i(q)(1-q) \tag{52}$$

for the susceptibility $\chi_i^{(1)}$.

Let us consider an ageing system and add the perturbation $\Delta H_i^{(1)}$ at time t_w. We have

$$\chi_i^{(1)}(t) = \lim_{\delta\to 0} \left\langle \Delta H_i^{(1)} \right\rangle_t = \frac{\sum_{a=1,M} R_i^a(t_w, t)}{M}. \tag{53}$$

However, in the limit small ϵ in the dynamics does not depend on the clone a so that we get

$$\chi_i^{(1)}(t_w, t) = R_i(t_w, t). \tag{54}$$

Assuming that

$$\lim_{t\to\infty} \chi_i^{(1)}(t_w, t) = \chi_i^{(1)} \tag{55}$$

we arrive at

$$\lim_{t\to\infty} R_i(t_w, t') = \chi_i^{(1)} = \beta \int \mathrm{d}q\, P_i(q)(1-q). \tag{56}$$

We can now copy *mutatis mutandis* the proof of the usual FDR. For example let us define

$$\Delta H_i^{(2)} \equiv \sum_{a=1,M,b=1,M} h^{a,b} \sigma_i^a \sigma_i^b \tag{57}$$

where the variables h are Gaussian random variables with zero average and variance $\delta/(M)$. The static susceptibility is given by

$$\frac{\left\langle \Delta H_i^{(2)} \right\rangle}{M} = \delta\beta \sum_{a,b=1,M} \frac{1 - \left\langle \sigma_i^a \sigma_i^b \right\rangle^2)}{M} = \delta\beta \int \mathrm{d}q\, P_i(q)(1-q^2). \tag{58}$$

If we assume for simplicity a Langevin type of evolution, from the same steps of [10] we have

$$\chi_i^{(2)}(t_w, t) = 2 \int_{t_w}^{t} \mathrm{d}t'\, C_i(t', t) \frac{\partial R_i(t', t)}{\partial t'}. \tag{59}$$

For simplicity we assume that for large t', $R_i(t', t)$ becomes a function of only $C_i(t', t)$ (this assumption is not necessary; indeed, using the formula for all the moments it is possible to prove that equation (62) is automatically correct) and get

$$\lim_{t\to\infty} \chi_i^{(2)}(t_w, t) = 2 \int \mathrm{d}C\, C X_i(C) \tag{60}$$

where we have defined

$$X_i(t')(C(t', t)) = \frac{\partial R_i(t', t)}{\partial t'} \left(\frac{\partial C_i(t', t)}{\partial t'} \right)^{-1} \tag{61}$$

and

$$X_i(C) = \lim_{t'\to\infty} X_i(t')(C). \tag{62}$$

If also in this case the limits $t \to \infty$ and $\delta \to 0$ are exchanged, we get

$$\lim_{t\to\infty} \chi_i^{(2)}(t_w, t) = \chi_i^{(2)} = \beta \int \mathrm{d}q\, P_i(q)(1-q^2). \tag{63}$$

Generalizing the previous arguments, we get

$$\chi_i^{(s)}(t) = s\int_{t_w}^{t} \mathrm{d}t'(C_i(t',t)^{s-1})\frac{\partial R_i(t',t)}{\partial t'} = s\int \mathrm{d}C\, C^{s-1}X_i(C)$$
$$= \beta\int \mathrm{d}q\, P_i(q)(1-q^s) = s\beta\int x_i(q)q^{s-1}. \tag{64}$$

We thus arrive at the conclusion that

$$X_i(C) = x_i(C) \equiv \int_0^C \mathrm{d}q\, P_i(q)\,\mathrm{d}q. \tag{65}$$

This is the local relation among the static and the local FDR.

A few remarks are in order:

- If we take a sequence of larger and larger systems, the dynamical quantities converge to a well-defined limit when the volume goes to infinity. Therefore, also the local overlap distribution $P_i(q)$ goes to a limit when the volume goes to infinity: this property implies that $P_i(q)$ depends only on the local environment (i.e. the couplings J not too far from the point i). It is clear that all the problems in defining the function $P_J(q)$ in the infinite volume limit, due to the difficulties in defining the equilibrium states in the infinite volume limit, fade away because the function $P_i(q)$ is not sensitive to the far away couplings or to the boundary conditions. It is quite possible that the construction we have presented in this paper is useful in a rigorous approach.
- The way in which we have coupled together our M clones is irrelevant. The only task they do is to correlate the states of one clone with the states of another clone and this may be implemented by any random coupling.

5.3. Bilocal perturbations

We are now ready to prove the thermometric indistinguishability of the sites. We consider two far away sites i and k and apply a perturbation that depends on both the spins at i and the spins at k.

A typical example is

$$\Delta H_{i,k}^{(3,2)} \equiv \sum_{a_1,a_2,a_3,b_1,b_2=1,M} h^{a_1,a_2,a_3,b_1,b_2}\,\sigma_i^{a_1}\sigma_i^{a_2}\sigma_i^{a_3}\sigma_k^{b_1}\sigma_k^{b_2}. \tag{66}$$

where the variables h are Gaussian random variables with zero average and variance δ/M^4. In the same way as before, we get

$$\lim_{\delta\to 0}\frac{\left\langle \Delta H_{i,k}^{(s_i,s_k)}\right\rangle}{\delta\beta M} \equiv \chi_{i,k}^{(s_i,s_k)} = \beta\int \mathrm{d}q_i\,\mathrm{d}q_k\, P(q_i,q_k)(1-(q_i)^{s_i}(q_k)^{s_k}) \tag{67}$$

where $P(q_i,q_k)$ is the probability distribution of q_i and q_k. If we compute the same quantity in the ageing regime for very large time, we have that the same quantity must be equal to the large time limit of

$$\int_{t_w}^{t}\mathrm{d}t\left[X_i(t)\frac{\partial C_i^{s_i}}{\partial t}C_k^{s_k} + X_k(t)C_i^{s_i}\frac{\partial C_k^{s_k}}{\partial t}\right]. \tag{68}$$

Imposing that the two expressions can be equal for arbitrary s_i and s_k, one recovers the form of $P(q_i,q_k)$ in terms of $X_i(t',t)$ and $X_k(t',t)$. By imposing that $P(q_i,q_k)$ is positive (i.e. it does not contain a derivative of a delta function), we find that in the region of large t' one must have

$$X_i(t',t) = X_k(t',t). \tag{69}$$

We finally obtain

$$P(q_i, q_k) = \int_0^1 \mathrm{d}x\, \delta(q_i - q_i(x))\delta(q_k - q_k(x)) \tag{70}$$

where $q_i(x)$ is the inverse function of $x_i(q)$.

The first equation is just thermometric indistinguishability of the sites during ageing while the second equation has some interesting consequences that will be investigated elsewhere.

We note that the probability distribution of the local overlap $P_i(q_i)$, being related to a dynamical quantity, must depend only on the local environment around the point i and therefore it must have a straightforward limit when the volume of the system goes to infinity (e.g. it should be independent of the boundary conditions). It is remarkable that for far away points (i, k) the two probability distributions $P_i(q_i)$ and $P_k(q_k)$ are independent of each other, but the joint probability distribution of q_i and q_k does not factorize as shown by equation (70), i.e. $P(q_i, q_k) \neq P(q_i)P(q_k)$. In some sense equation (70) tells us that only the part of $P(q_i, q_k)$ that corresponds to a given temperature does factorize.

6. Computing the local overlap distribution

The formulae presented in the previous section are useful in defining the local overlap and its distribution, but they are not handy as far as practical computations are concerned. The distribution $P_i(q)$ has rounded delta functions for finite M. On the other hand, the burden of the numerical computation increases violently with M. The best thermalization method (the parallel tempering) becomes slower and slower when the volume of the system (NM) increases. It is convenient to obtain this kind of information using different methods.

6.1. Computing the moments

In many cases knowledge of the first few moments of the function $P_i(q)$ is enough to compute this function. This approach is very useful in the case where we have some *a priori* information on the shape of this function. For example, in the case of one step broken replica symmetry, knowledge of the first two moments and m completely determines the function $P_i(q)$.

In other models, where the replica symmetry is broken in a continuous way, it is possible that a good approximation is given by simple expressions such as

$$P_i(q) = \theta\big(q - q_i^{EA}\big)(1-q)^{-1/2}Q_i(q) + m\delta\big(q - q_i^{EA}\big) \tag{71}$$

where $Q_i(q)$ is a low degree polynomial. However, the validity of similar formulae may strongly change from model to model.

The computation of the low moments is not computationally heavy. We consider the Gibbsian system with fixed M. If σ and τ are two equilibrium configurations of the model, it is possible to prove that

$$\overline{\Big\langle \prod_{a=1,M} \sigma_i^a \tau_i^a \Big\rangle} = q_i^{(M)} \equiv \int \mathrm{d}q\, q^M P_i(q) \tag{72}$$

or more generally

$$\overline{\big\langle \sigma_i^{a_1}\tau_i^{a_1} \ldots \sigma_i^{a_k}\tau_i^{a_k} \big\rangle} = q_i^{(k)} \tag{73}$$

where the indices a are arbitrary as soon as they are all different (obviously $k < M$).

The proof of the previous relation can be obtained by computing the local susceptibilities such as equation (58). Let us consider the case $k = 2$. We could also define

$$\Delta H_i^{(2)} = \left(\frac{M}{M-1}\right)^{1/2} \sum_{a=1,M,b=1,M;a\neq b} h^{a,b}\,\sigma_i^a \sigma_i^b. \tag{74}$$

The corresponding susceptibility does not depend on M and it is trivially equal to the previously defined susceptibility when $M \to \infty$. In other words

$$\overline{\langle \sigma_i^a \tau_i^a \sigma_i^b \tau_i^b \rangle} \tag{75}$$

does not depend on M, a and b as soon as $a \neq b$.

This result can be checked in an explicit way in the two explicit cases discussed before, equations (2) and (4).

The computation of the moment of order k involves only the thermalization of a system with only kN sites and it can be done without too much computational effort for not too large k.

6.2. Introducing a state reservoir

Another possibility, which is more interesting especially for analytic computations, consists in noting that the local overlap distribution depends only on the local environment. Therefore, we can embed the local environment in a large system, and the local properties should not be related to that of the rest of the system as soon as the rest of the system has the same distribution of states as the original system.

Let us consider a simple case: a spin glass model in three dimensions. Let us suppose that within the required accuracy, knowledge of the couplings up to a distance R from the site i determines the function $P_i(q)$. Let us consider a system of size $L > 4R$ with the same probability distribution of the couplings, with the constraint however that there are two points i_1 and i_2 such that the local environments of radius R around each of these two points (that are at distance greater than $2R$) are the same and equal to that of the original system at the point i.

In this case the same argument as before leads to the conclusions that if we take two different equilibrium configurations σ and τ, we have that

$$\overline{\langle \sigma_{i_1} \sigma_{i_2} \tau_{i_1} \tau_{i_2} \rangle} = q_i^{(2)} \tag{76}$$

where the bar denotes the average over the couplings that do not belong to the fixed environment.

A different possibility would be to take two copies of the local environment and couple the spins at the boundary of each copy to other spins that remain on a Bethe lattice. If the state distribution of the Bethe lattice is the same as that of the three-dimensional lattice, one should get identical results.

In both cases the main role of the system outside the local environment was to put the local environment in all different possible states, with the correct probability distribution. In a nutshell it played the role of a state reservoir. As long as the detailed nature of this reservoir is not important, we can consider the simplest possible model for it. A very convenient choice is the following: we model the interaction of the local environment with the rest of the world by introducing some extra fields h and an extra term in the Hamiltonian given by

$$\sum_{k\in S} h_k \sigma_k \tag{77}$$

where the sum is done over the spins of the surface (S). The free energy corresponding to a given choice of the variables h is given by $F[h]$ and the corresponding magnetization of the site i is $m[h]$.

We should now model the ensembles of h and w. We should introduce the distribution of the weight w_α and for each state α, we should give the set of h.

Let us see how this can be done and consider only the simplest case: a system with one step replica symmetry where we fix the value of m. In this case we can assume that w are distributed according to the expression in equation (21). We can assume the variable h^α has a given probability distribution, e.g. one of the simplest possibilities is that

$$h_k^\alpha = \overline{h}_k + \delta h_k^\alpha \tag{78}$$

where both $\overline{h}_k$ and δh_k^α are uncorrelated random variables with zero average and variance $\Delta h^{(0)}$ and $\Delta h^{(1)}$. Different forms of the probability distribution of the fields h can be used; however, if the local environment is sufficiently large, the result should not depend on the form of this probability distribution or the variances $\Delta h^{(0)}$ and $\Delta h^{(1)}$.

One finally finds that the two parameters that identify the probability distribution, i.e. q_i^0 and q_i^1, are given respectively by

$$\begin{aligned} q_i^0 &= \left(\overline{\frac{\sum_{\alpha=1,\mathcal{A}} w_\alpha \exp(-\beta F[h^\alpha]) m[h^\alpha]}{\sum_{\alpha=1,\mathcal{A}} w_\alpha \exp(-\beta F[h^\alpha])}} \right)^2 \\ q_i^1 &= \overline{\frac{\sum_{\alpha=1,\mathcal{A}} w_\alpha \exp(-\beta F[h^\alpha]) m[h^\alpha]^2}{\sum_{\alpha=1,\mathcal{A}} w_\alpha \exp(-\beta F[h^\alpha])}} \end{aligned} \tag{79}$$

where $\mathcal{A}$ is a large number, the bar denotes the average over the magnetic field h and the weights w. Using the techniques of [24], the average over w can be done and one obtains

$$\begin{aligned} q_i^0 &= \left(\overline{\frac{\sum_{\alpha=1,\mathcal{A}} \exp(-\beta m F[h^\alpha]) m[h^\alpha]}{\sum_{\alpha=1,\mathcal{A}} \exp(-\beta m F[h^\alpha])}} \right)^2 \\ q_i^1 &= \overline{\frac{\sum_{\alpha=1,\mathcal{A}} \exp(-\beta m F[h^\alpha]) m[h^\alpha]^2}{\sum_{\alpha=1,\mathcal{A}} \exp(-\beta m F[h^\alpha])}}. \end{aligned} \tag{80}$$

Of course, if our model is a Bethe lattice, the computations are quite simple and we reobtain the results of [24], i.e. a local version of equations [25].

7. Conclusions

The main results of this paper are the definition of a local probability distribution of the overlap, which depends on the site for a fixed sample. The properties of this local probability distribution are related to the local fluctuation dissipation relations, which automatically follow from the present formalism. The property of thermometric indistinguishability of the sites turns out to be a byproduct of our approach: during the ageing regime all the sites are characterized by the same effective temperature.

The two time local correlation functions can be written as

$$C_{i,}(t_w, t) = C_i(x(t_w, t)) \tag{81}$$

where the function $C_i(x)$ is the inverse of the function $x_i(C)$ and can be obtained only by static measurements. The whole local dependence of off-equilibrium correlations and responses can be computed from static quantities. The only quantity that cannot be computed from equilibrium consideration is the global effective temperature as a function of the two times, i.e. $\beta x(t_w, t)$.

The physical scenario that emerges is quite clear. The local overlap distribution can be defined by averaging over the ensemble of the states of the system, and the introduction of the Gibbs system is a technical tool for performing this average in a constructive way. During the dynamics the system locally explores different states of the system in a random way. It should not be surprising that the dynamical average is deeply connected to the static average on all possible different states.

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Appendix

It is interesting to consider what happens if we look at the model with Hamiltonian

$$\sum_{a=1,M} H(\sigma^a) + \epsilon \sum_{a,b=1,M;i,j=1,N} K_{i,j}^{a,b}\, \sigma_i^a \sigma_j^b \tag{82}$$

in the limit $M \to \infty$ but with fixed N. In this case the second part of the Hamiltonian coincides with that of the Sherrington–Kirkpatrick model.

For small enough ϵ one finds that the solution of the model is the replica symmetric one. It depends on a parameter q, which can be found from the solution of the equations:

$$q = \frac{\sum_{i=1,N} \overline{m_i[h]^2}}{N} \tag{83}$$

where the magnetization are obtained by considering the statistical average for one sample with Hamiltonian

$$H(\sigma) + \sum_i h_i \sigma_i \tag{84}$$

and the magnetic fields are random independent identically distributed Gaussian variables with zero average and variance ϵq and the bar denotes the average over the fields h.

However, by increasing ϵ this solution may become unstable (De Almeida Touless line) and at higher ϵ one has to look for replica broken solutions. A computation similar to the original one shows that replica broken phase happens when $\epsilon^2 N = O(1)$ and this result is at the origin of the condition $\epsilon^2 N \gg 1$.

References

[1] Marinari E, Parisi G, Ricci-Tersenghi F and Ruiz-Lorenzo J J 1998 *J. Phys. A: Math. Gen.* **31** L481
Marinari E, Parisi G and Ruiz-Lorenzo J 1998 Numerical simulations of spin glass systems *Spin Glasses and Random Fields* ed P Young (Singapore: World Scientific)
Marinari E, Parisi G, Ricci-Tersenghi F, Ruiz-Lorenzo J and Zuliani F 2000 *J. Stat. Phys.* **98** 973
[2] Parisi G 2002 Local fluctuation dissipation relations *Preprint* cond-mat/0208070
[3] Cugliandolo L F and Kurchan J 1993 *Phys. Rev. Lett.* **71** 173
Cugliandolo L F and Kurchan J 1994 *J. Phys. A: Math. Gen.* **27** 5749
[4] Franz S and Mézard M 1994 *Europhys. Lett.* **26** 209
[5] Cugliandolo L F, Kurchan J and Parisi G 1994 *J. Physique I* **4** 1641
[6] For a review see Bouchaud J P, Cugliandolo L F, Kurchan J and Mézard M 1997 Out of equilibrium dynamics in spin glasses and the other glassy systems *Spin Glasses and Random Fields* ed P Young (Singapore: World Scientific)

[7] Guerra F 1997 *Int. J. Phys.* B **10** 1675
[8] Aizenman M and Contucci P 1998 *J. Stat. Phys.* **92** 765
[9] Parisi G 1998 On the probabilistic formulation of the replica approach to spin glasses *Preprint* cond-mat/9801081
[10] Franz S, Mézard M, Parisi G and Peliti L 1998 *Phys. Rev. Lett.* **81** 1758
Franz S, Mézard M, Parisi G and Peliti L 1999 *J. Stat. Phys.* **97** 459
[11] Franz S and Rieger H 1995 *J. Stat. Phys.* **79** 749
[12] Parisi G 1997 *Phys. Rev. Lett.* **78** 4581
[13] Kob W and Barrat J-L 1997 *Phys. Rev. Lett.* **79** 3660
[14] Grigera T S and Israeloff N E 1999 *Phys. Rev. Lett.* **83** 5038
[15] Bellon J and Ciliberto S 2002 *Preprint* cond-mat/0201224 (2003 *Physica* D at press)
[16] Hérisson D and Ocio M 2002 *Phys. Rev. Lett.* **88** 257202
[17] Kennett M P and Chamon C 2001 *Phys. Rev. Lett.* **86** 1622
Kennett M P, Chamon C and Ye J 2001 *Phys. Rev.* B **64** 224408
[18] Castillo H E, Chamon C, Cugliandolo L and Kennett M 2002 *Phys. Rev. Lett.* **88** 237201
Castillo H E, Chamon C, Cugliandolo L and Kennett M 2002 *Phys. Rev. Lett.* **89** 217201
[19] Montanari A and Ricci-Tersenghi F 2002 A microscopic description of the aging dynamics: fluctuation–dissipation relations, effective temperature and heterogeneities *Preprint* cond-mat 0207416
[20] Poole P H *et al* 1997 *Phys. Rev. Lett.* **78** 3394
Barrat A and Zecchina R 1999 *Phys. Rev.* E **59** R1299
Ricci-Tersenghi F and Zecchina R 2000 *Phys. Rev.* E **62** R7567
[21] Cugliandolo L, Kurchan J and Peliti L 1997 *Phys. Rev.* E **55** 3898
[22] Franz S and Virasoro M A 2000 *J. Phys. A: Math. Gen.* **33** 891
[23] Mézard M and Virasoro M A 1985 *J. Physique* **46** 1293
[24] Mézard M and Parisi G 2001 *Eur. Phys. J.* B **20** 217
Mézard M and Parisi G 2002 The cavity method at zero temperature *Preprint* cond-mat/0207121 (2003 *J. Stat. Phys.* at press)
[25] Mézard M, Parisi G and Zecchina R 2002 *Science* **297** 812